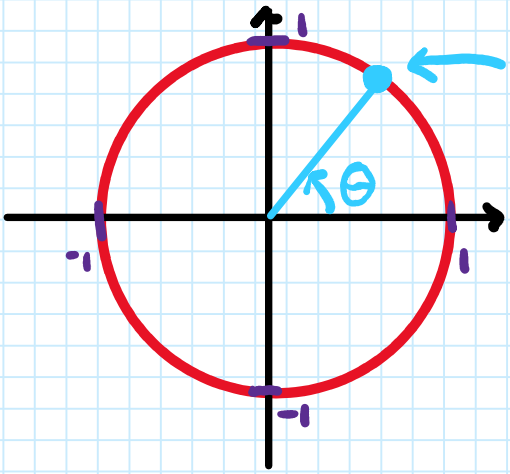
6/23 Precalc

The Unit Circle Circle of radius 1 centered at origin

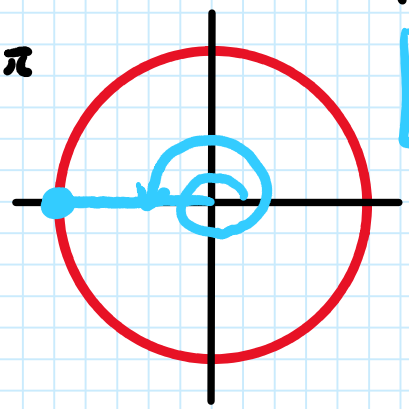


← "terminal point determined by θ"

Def The terminal point of angle θ is the point that the terminal side of θ intersects the unit circle

Ex Find the terminal point of the angle:

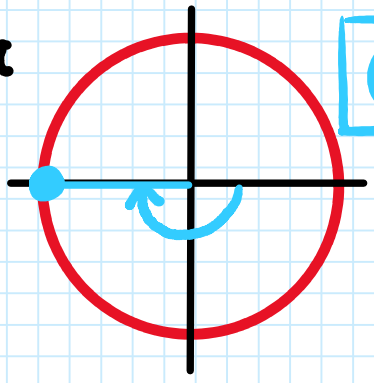
(a) 3π



$(-1,0)$

note: π and 3π are coterminal so their terminal points are the same

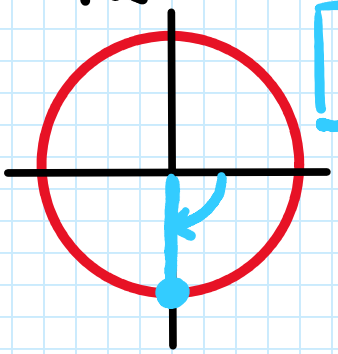
(b) $-\pi$ ← clockwise



$(-1,0)$

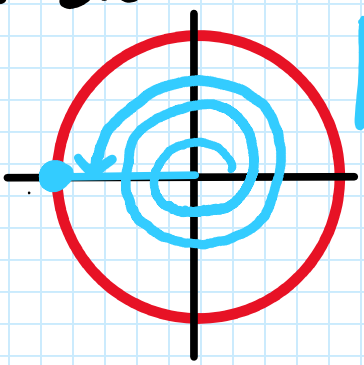
note: $-\pi$, π, 3π are all coterminal

(c) $-\pi/2$



$(0,-1)$

(d) 5π



$(-1,0)$

More generally:

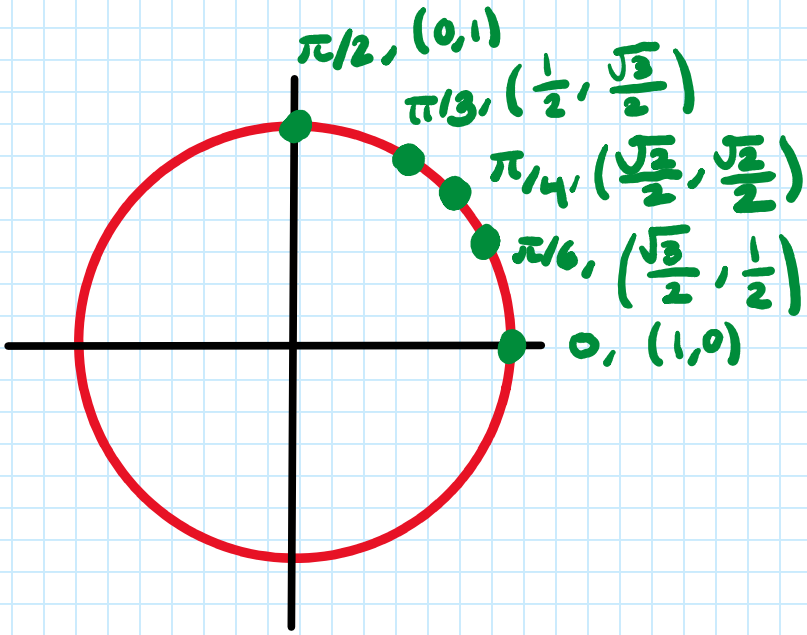
- even integer multiples of π have terminal point $(1,0)$

  $0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$

- odd integer multiples of π have terminal point $(-1,0)$

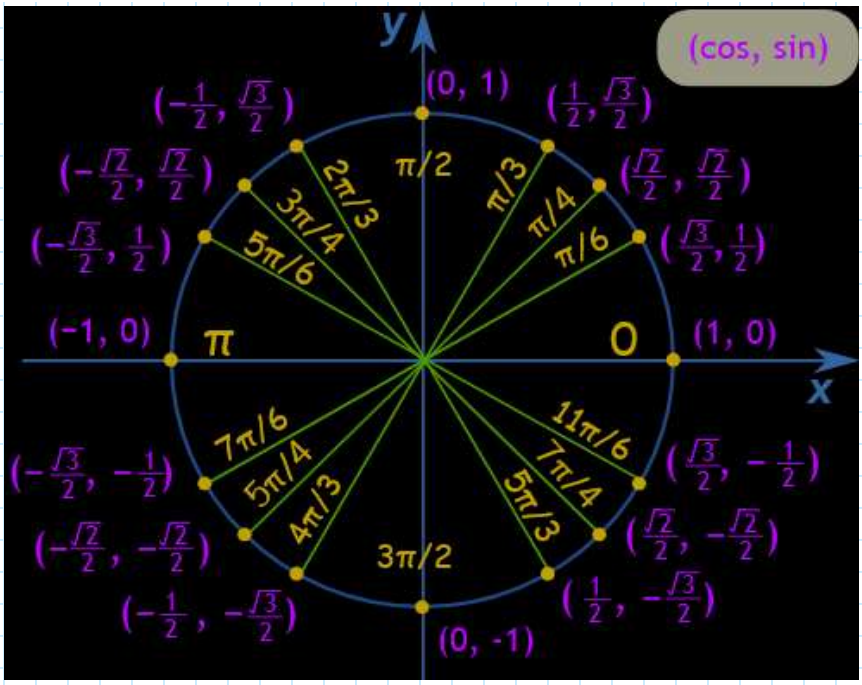
  $\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, \ldots$

Special Terminal Points



← These are the important ones to memorize.

The rest can be found through symmetry of the unit circle



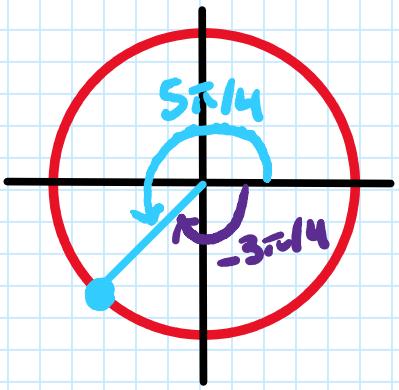
source mathisfun.com

Find terminal points of

(a) $3\pi/4$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ ← just read unit circle

(b) $7\pi/6$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ ← read unit circle

(c) $-\frac{3\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ ← not explicitly on unit circle but note $-\frac{3\pi}{4}$ and $\frac{5\pi}{4}$ are coterminal



★ $-\frac{3\pi}{4} = \frac{5\pi}{4} - 2\pi$

(d) $-\frac{\pi}{3}$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ ← $5\pi/3$ and $-\pi/3$ are coterminal

★ $-\frac{\pi}{3} = \frac{5\pi}{3} - 2\pi$

General fact Adding/subtracting integer multiples of

General fact Adding/subtracting integer multiples of 2π gives coterminal angles ★